Math 220 - Discrete Mathematics - Fall 2016

Homework 13b

ATTENTION: The handout from class uses $\vee$ instead of $+$ and $\wedge$ instead of $\cdot$ in its definition of an abstract Boolean algebra.

1. Handout Problems

Section 12.1 # 1, 2, 3a, 4a, 35, 37, 38, 39

In #39, you only need to do $\overline{x+y} = \bar{x} \cdot \bar{y}$ since we did the other one in class.

2. Additional Problems

(1) Prove the Absorption laws

$$\begin{aligned} x + x \cdot y &= x \\ x(x+y) &= x \end{aligned}$$

(2) Let $n \in \mathbb{N}$ be a number which is "square-free," that is, n cannot be written in the form $n = ab^2$ for $a, b \in \mathbb{N}$. Define $B = \{x \in \mathbb{N} | \, x|n\}$, the set of positive divisors of n. Using 1 as the 0 element and n as the 1 element, for $x, y \in B$, if we define $x + y = \gcd(x, y)$, $x \cdot y = \operatorname{lcm}(x, y)$, and $\bar{x} = \dfrac{n}{x}$, then we have a Boolean algebra. Prove that B with these operations satisfies all the properties of a Boolean algebra.

(3) Give a counter example to the above structure being a Boolean algebra if n is not square-free, e.g., if $n = 12$.